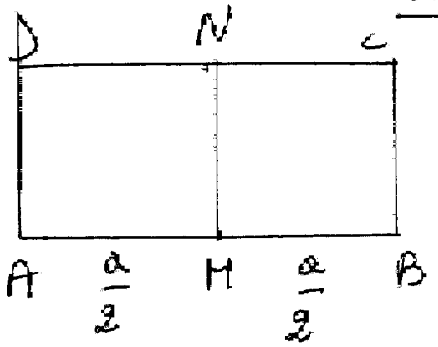
QUESITO 2

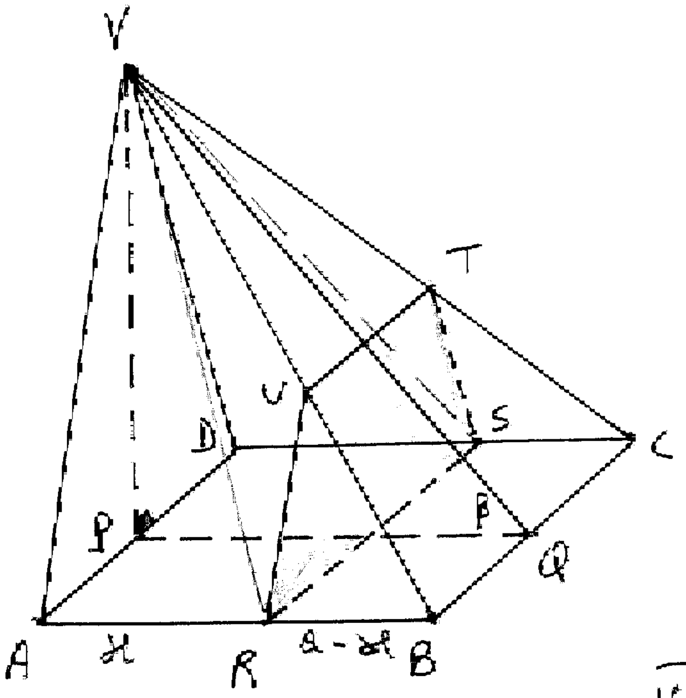


a) $\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AD}}{\overline{AM}}$

......

$$\left[\ \overline{AD} = \frac{a}{\sqrt{2}} = \frac{a\sqrt{2}}{2}\ \right]$$

b)



$\cos\beta = \frac{2}{\sqrt{13}}$

$\sin\beta = \frac{3}{\sqrt{13}}$

$V = \frac{1}{3}\, a(ABCD) \cdot \overline{VP}$

$\overline{VP} = \overline{PQ}\,\mathrm{tg}\,\beta =$

$= \boxed{\frac{3}{2}a}$

$$V = \frac{1}{3}\, a\left(a\frac{\sqrt{2}}{2}\right)\left(\frac{3}{2}a\right)$$

$$\boxed{V = \frac{\sqrt{2}}{4}a^3}$$

c) $0 < x < a$

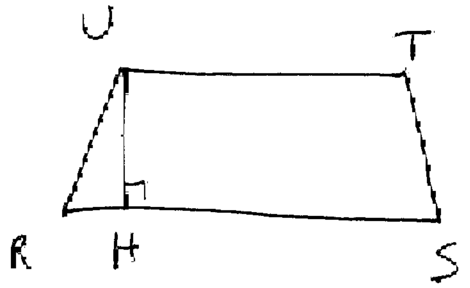
$$\overline{AV} = \sqrt{\overline{VP}^2 + \overline{AP}^2} = \cdots = \sqrt{\frac{19}{8}a^2}$$

$$\overline{VB} = \sqrt{\overline{AV}^2 + \overline{AB}^2} = \sqrt{\frac{27}{8}a^2}$$

$$\overline{RU} : \overline{AV} = \overline{RB} : \overline{AB} \Rightarrow \overline{RU} = \frac{1}{2}\sqrt{\frac{15}{2}}(a - x)$$

$$\overline{BU} = \sqrt{\overline{RU}^2 + \overline{RB}^2} = \dots = \frac{3}{2}\sqrt{\frac{3}{2}}(a - x)$$

$$\left.\begin{array}{l}\overline{TU} : \overline{BC} = \overline{VU} : \overline{VB} \\ \qquad\qquad\quad \Downarrow \\ \qquad\quad \overline{VB} - \overline{UB}\end{array}\right\} \Rightarrow \overline{TU} = \frac{\sqrt{2}}{2}x$$



$$\overline{RH} = \frac{1}{2}\left(\frac{a\sqrt{2}}{2} - \frac{x\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}(a - x)$$

$$\overline{UH} = \sqrt{\overline{RU}^2 - \overline{RH}^2} = \dots = \frac{3}{2}(a - x)$$

$$\mathcal{A}(RSTU) = \frac{(\overline{RS} + \overline{UT}) \cdot \overline{UH}}{2}$$

$$\boxed{\mathcal{A} = \frac{3}{8}\sqrt{2}(a^2 - x^2)}$$

d) Il solido ARSTUV si può scomporre nella piramide 1 di base ARSD e vertice V e nella piramide 2 di base RSTU e vertice V (con altezza uguale ad x):

$$V_1 = \frac{1}{3}\,\mathcal{A}(ARSD)\cdot\overline{VP} = \frac{\sqrt{2}}{8}a^3$$

$$V_2 = \frac{1}{3}\,\mathcal{A}(RSTU)\cdot\overline{AR} = \dots = \frac{3\sqrt{2}}{64}a^3$$

$$\boxed{V_1 + V_2 = \frac{11}{64}a^3\sqrt{2}} \quad (1^\circ \text{ solido})$$

Il volume del secondo solido si ottiene per differenza tra la piramide ABCDV ed il 1° solido:

$$V = \frac{a^3\sqrt{2}}{4} - \frac{11}{64}a^3\sqrt{2} = \boxed{\frac{5}{64}a^3\sqrt{2}}$$

N.B. Il volume del secondo solido poteva essere calcolato mediante l'integrale

$$\int_{\frac{a}{2}}^{a} S(x)\,dx = \int_{\frac{a}{2}}^{a} \frac{3}{8}\sqrt{2}\,(a^2 - x^2)\,dx =$$

$$= \frac{3}{8}\sqrt{2}\left[a^2x - \frac{x^3}{3}\right]_{\frac{a}{2}}^{a} = \frac{5}{64}a^3\sqrt{2}$$